Probability

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Contents

Probability

Probability is the **likeliness** of occurring any event(s).

Some related terms

Deterministic experiment Vs **Random experiment:**

An experiment whose outcome is predictable in advance is called deterministic experiment. Everyone conducting that experiment will get the same outcome.

An experiment whose outcome is not predictable with certainty in advance is called a random experiment. If a random experiment is performed then one of many possible outcomes will occur.

Some related terms

Deterministic experiment Vs **Random experiment:**

Example:

Deterministic experiment:

- Measuring linear distance from Dhaka to Chittagong.
- Measuring length of a scale.

Random experiment:

- Measuring weight of a person at different times.
- Tossing a coin.

Some related terms

Sample Space:

The set of all possible outcomes of a random experiment is called sample space of that experiment and is denoted by S. Each individual outcome is called a sample point.

For example;

throwing a dice- $S = \{1, 2, 3, 4, 5, 6\}$

Lifetime of a lightbulb- $S = \{x | 0 \le x < \infty\} = [0, \infty)$

Some related terms

Event:

Any subset E of a sample space S is an event.

For example;

Dice throw experiment- S= {1, 2, 3, 4, 5, 6 }

Say number 2 turned up in a throw. Then we will say, event E= {2} has occurred.

Some related terms

Mutually Exclusive events:

Two events are called mutually exclusive if both the events cannot occur simultaneously in a single trial. In other words, if one of those events occurs, the other event will not occur.

For example;

in a trial of coin toss experiment, event E1= {Head} and event E2={tail} will not occur simultaneously. So, E1 and E2 are mutually exclusive events.

On a day, Event E1={Rain} & event E2={Sunny} may occur simultaneously. These are not mutually exclusive events.

Some related terms

Collectively exhaustive events:

Collectively exhaustive events are those, which includes all possible outcomes.

For example;

In a coin tossing experiment events E_1= {Head} and event E_2={tail} are collectively exhaustive, because together they comprise the all the outcomes that are possible in a coin tossing experiment. There are no other possible outcomes of this experiment than these two.

Some related terms

Equally likely events:

The events of a random experiment are called equally likely if the chance of occurring those events are all equal.

For example;

In a coin tossing experiment, the events $E1= \{Head\}$ and event $E2=\{tail\}$ are equally likely, because the chance of occurring $E1$ is as same as occurring $E2$.

On a day, Event $E1=\{Rain\}$ & event $E2=\{No\ rain\}$ may not be equally likely.

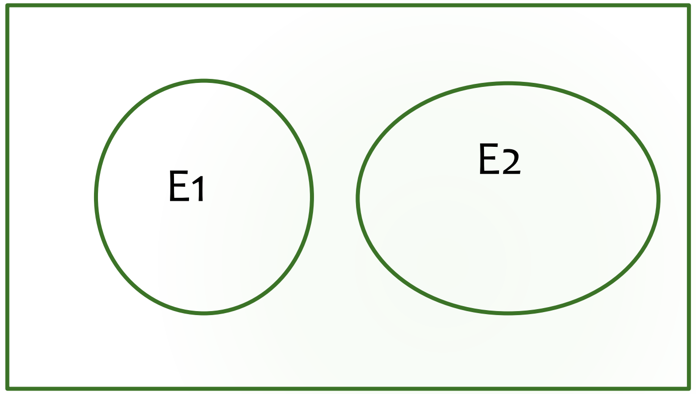
Some related terms

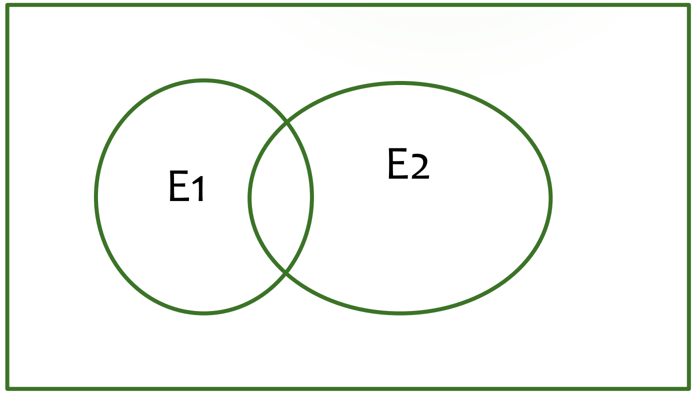
Disjoint events:

Two events are called disjoint, if they have no common elements between them.

Mutually exclusive events are disjoint events.

Disjoint events





Joint events

Approaches of assigning probability

- At first we identify the sample space S of the random experiment.
- We then define our favorable event and assign probability to the event using one of the following 3 basic approaches-
  - Classical approach
  - Frequency approach
  - Subjective approach

Approaches of assigning probability

Classical approach:

(when the outcomes are equally likely, mutually exclusive and collectively exhaustive)

- If the sample space of a random experiment has a finite number (n_s) of outcomes
- n_E of these outcomes are favorable to an event E

Then, the probability of occurring event E, denoted by P(E) is-

$$P(E) = \frac{n_E}{n_S}$$

Approaches of assigning probability

Classical approach:

For example;

Dice throwing experiment-

S= {1, 2, 3, 4, 5, 6}

Consider two events, E1= {2} and E2= {2, 4, 6}

Here, n_{E1}= 1 and n_{E2}= 3. Also, n_S= 6

Therefore, probability of occurring event E1 is, $P(E_1) = \frac{n_{E_1}}{n_S} = \frac{1}{6}$

probability of occurring event E2 is, $P(E_2) = \frac{n_{E_2}}{n_S} = \frac{3}{6} = \frac{1}{2}$

Approaches of assigning probability

Frequency approach:

If an experiment is repeated n times under the same conditions and event E occurs f times out of n times, then

$$P(E) = \lim_{n \to \propto} \frac{f}{n}$$

That is, when n is very large, P(E) is very close to the relative frequency of event E.

Approaches of assigning probability

Frequency approach:

For example;

In a dice throwing experiment- S= {1, 2, 3, 4, 5, 6}

And our favorable event is E= {2}

Let, 2 occurred a total of 998 times out of total 6000 trials.

Therefore $P(E) = \lim_{n \to \propto} \frac{998}{6000} \approx \frac{1}{6}$

Approaches of assigning probability

Subjective approach:

Based on the judgement (personal experience, prior information and belief etc.), one can assign probability to an event E of a random experiment.

For example; on a day of summer someone made a statement on probability that rain will occur on that day is .70, based on his previous experience.

Axioms of probability

Valid probabilities will follow 3 axioms-

Axiom 1: (Axiom of positivizes) : 0≤ P(E) ≤1

Axiom 2: (Axiom of certainty) : P(S) = 1

Axiom 3: (Axiom of additivity) : For a sequence of disjoint events E1, E2, … , En-

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$$

Example 1

In a community of 400 people, 20 people has a particular disease. If a person is selected randomly from that community, what is the probability that he/ she does not has the disease?

Example 1

In a community of 400 people, 20 people has a particular disease. If a person is selected randomly from that community, what is the probability that he/ she does not has the disease?

Solution:

Let, D= the randomly selected person has the disease

$$here, P(D) = \frac{f}{n} = \frac{20}{400} = .05$$
$$\therefore P(D^c) = 1 - .05 = 0.95$$

So, the probability that he/ she does not have the disease is 0.95

Probability Laws:

Addition Law

- **For disjoint events A and B-**

The probability that, either event A or event B will occur is,

$$P(A \cup B) = P(A) + P(B)$$

- **For disjoint events A, B, C, … , and Z-**

The probability that, either event A or event B or event C or … or event Z will occur is,

$$P(A \cup B \cup C \cup \cdots \cup Z) = P(A) + P(B) + P(C) + \ldots + P(Z)$$

Probability Laws:

Addition Law

- **For joint events A and B-**

The probability that, either event A or event B or both will occur is,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **For joint events A, B, and C**

The probability that, either event A or event B or event C or any two of them or all will occur is,

$$P(A \cup B \cup C)$$
$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C)$$
$$+ P(A \cap B \cap C)$$

Probability Laws:

Addition Law

Example 2:

In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both.

If an employee is selected randomly from that company, then

a. What is the probability that the employee has either motorcycle or private car?

b. What is the probability that the employee has neither motorcycle nor private car?

Probability Laws:

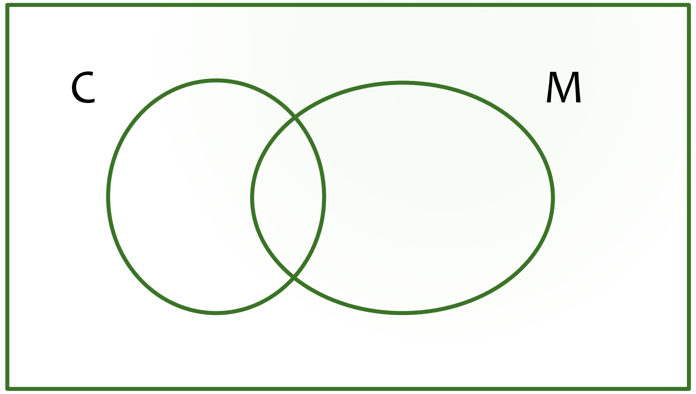
Addition Law

Solution:

Let,

M= the randomly selected employee has motorcycle

C= the randomly selected employee has car



Here, $P(M) = \frac{60}{100} = 0.6,\ P(C) = \frac{40}{100} = 0.4$

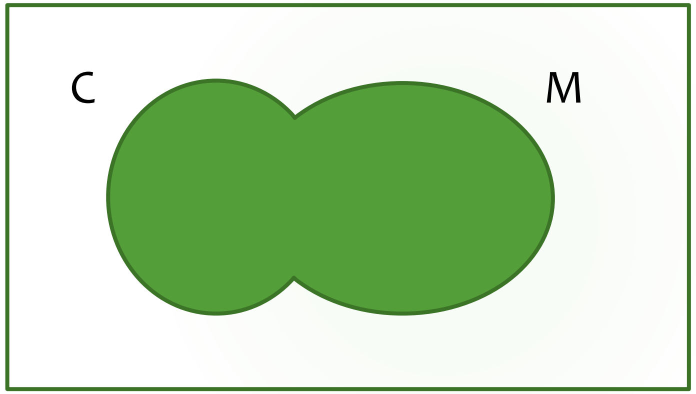
$$P(M \cap C) = \frac{20}{100} = 0.2$$

Probability Laws:

Addition Law

Solution(contd.)**:**

a. probability that the person has either motorcycle or private car is-
$P(M \cup C) = P(M) + P(C) - P(M \cap C)$
$= 0.6 + 0.4 - 0.2 = 0.8$



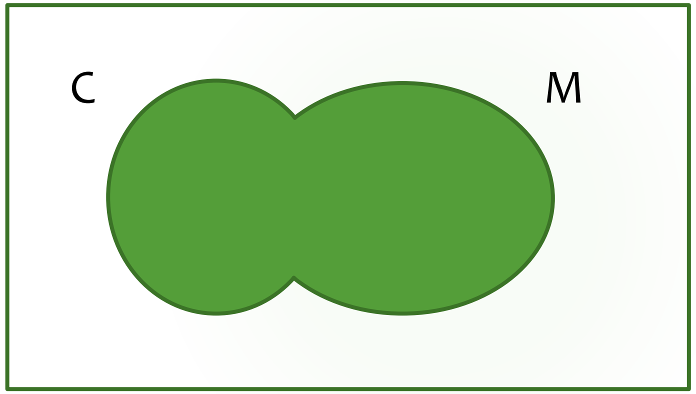
Probability Laws:

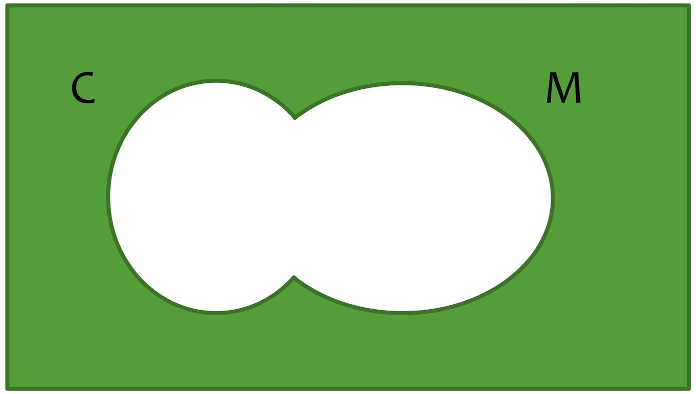
Addition Law

Solution(contd.)**:**

a. probability that the person has either motorcycle or private car is-
$P(M \cup C) = P(M) + P(C) - P(M \cap C)$
$= 0.6 + 0.4 - 0.2 = 0.8$

b. probability that the person has neither motorcycle nor private car is-
$P\left((M \cup C)^c\right) = 1 - P(M \cup C)$
$= 1 - 0.8 = 0.2$





Probability Laws:

Conditional Probability:

The conditional probability of an event E, given that another event F has already happened is,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \qquad for\ P(F) > 0$$

Probability Laws:

Conditional Probability:

Illustration: A dice throwing experiment.

S= {1, 2, 3, 4, 5, 6}

P(x)	Probability
P(1)	
P(2)	
P(3)	
P(4)	
P(5)	
P(6)	
Total	

Probability Laws:

Conditional Probability:

Illustration: A dice throwing experiment.

S= {1, 2, 3, 4, 5, 6}

P(x)	Probability
P(1)	1/6
P(2)	1/6
P(3)	1/6
P(4)	1/6
P(5)	1/6
P(6)	1/6
Total	**1**

Probability Laws:

Conditional Probability:

Illustration(contd.):

Now assume that, someone said that, in that throw someone said that, a odd number appeared on the dice.

That is, event F= {1, 3, 5} has happened.

So, our sample space S has reduced to S*= {1, 3, 5} = F

Probability Laws:

Conditional Probability:

Illustration(contd.):

Now, S*= {1, 3, 5}

Since, these outcomes were equally likely in the original sample space, they are equally likely in revised sample space

P(x|F)	Probability
P(1|F)	
P(2|F)	
P(3|F)	
P(4|F)	
P(5|F)	
P(6|F)	
Total	

Probability Laws:

Conditional Probability:

Illustration(contd.):

Now, S*= {1, 3, 5}

Since, these outcomes were equally likely in the original sample space, they are equally likely in revised sample space

P(x|F)	Probability
P(1|F)	1/3
P(2|F)	0
P(3|F)	1/3
P(4|F)	0
P(5|F)	1/3
P(6|F)	0
Total	**1**

Probability Laws:

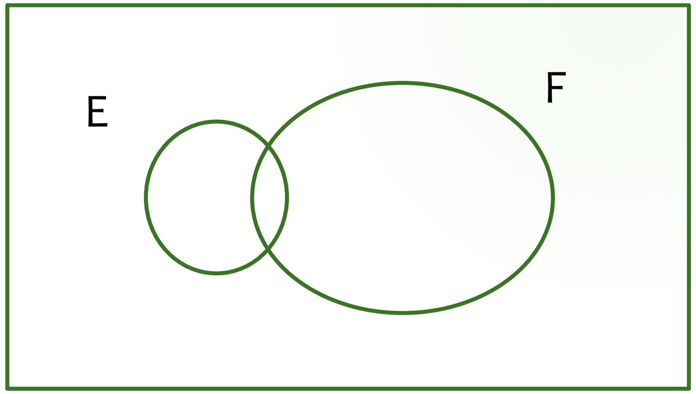
Conditional Probability:

Illustration(contd.):

Now, S*= {1, 3, 5}

Let, E= {1} and F= {1, 3, 5}

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



Probability Laws:

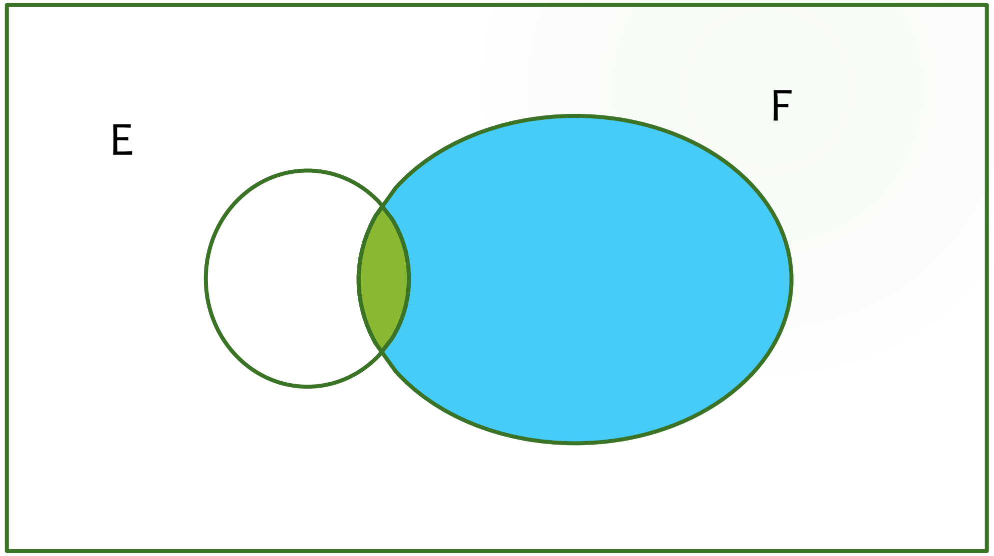
Conditional Probability:

Illustration(contd.):

Now, S*= {1, 3, 5}

Let, E= {1} and F= {1, 3, 5}

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$



Probability Laws:

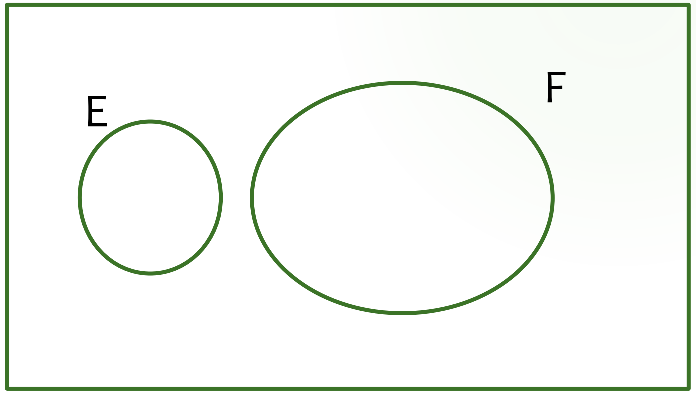
Conditional Probability:

Illustration(contd.):

Now, S*= {1, 3, 5}

Let, E= {2} and F= {1, 3, 5}

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{3}{6}} = 0$$



Probability Laws:

Example 3:

In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both.

If an employee is selected randomly from that company, then

a. What is the probability that the employee has a car?

b. If it is known that the employee has a motorcycle, then what is the probability that the employee also has a car?

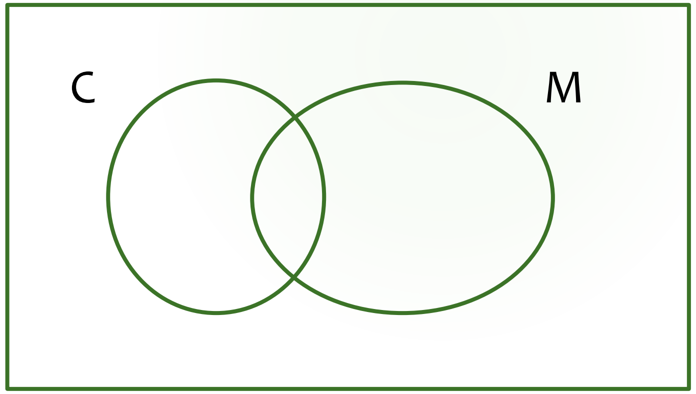
Probability Laws:

Solution:

Let,

M= the randomly selected employee has motorcycle

C= the randomly selected employee has car



Here, $P(M) = \frac{60}{100} = 0.6,\ P(C) = \frac{40}{100} = 0.4$

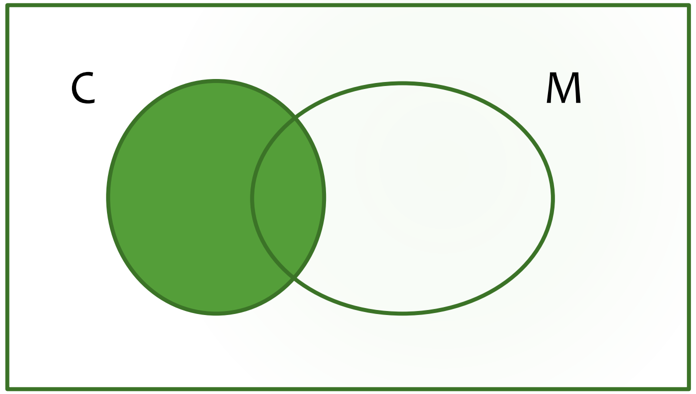
$$P(M \cap C) = \frac{20}{100} = 0.2$$

Probability Laws:

Solution(contd.):

a. probability that the employee has a car is-

$$P(C) = 0.4$$

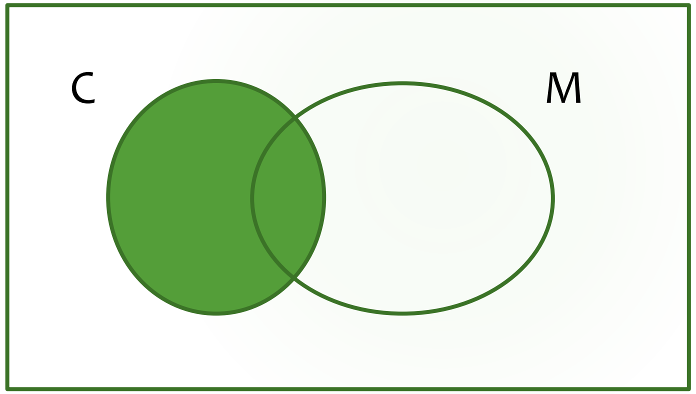


Probability Laws:

Solution(contd.)**:**

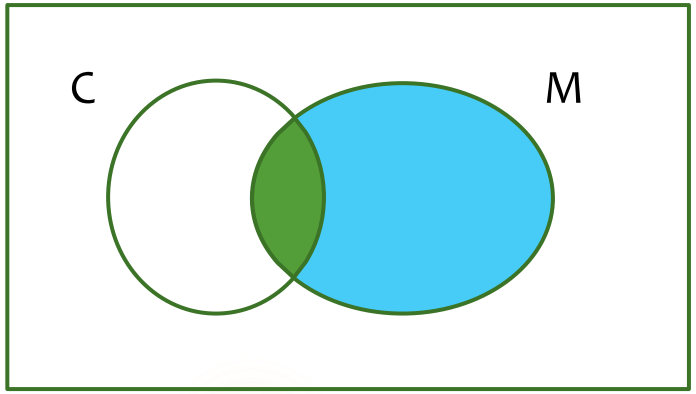
a. probability that the employee has a car is-

$$P(C) = 0.4$$



b. If it is known that the employee has a motorcycle, then what is the probability that the employee also has a car is,

$$P(C|M) = \frac{P(M \cap C)}{P(M)} = \frac{0.2}{0.6} = \frac{1}{3} = 0.33$$



Probability Laws:

Multiplication Law

- **For two dependent events E and F-**

The probability that, both event E and event F will occur simultaneously is,

$$P(E \cap F) = P(E|F)\, P(F)$$

Here, occurrence of event E depends on occurrence of event F.

- **For two independent events E and F-**

The probability that, both event E and event F will occur simultaneously is,

$$P(E \cap F) = P(E)\, P(F)$$

Probability Laws:

Multiplication Law

Example 4:

In rainy season, it rains 70% of the days in Bangladesh. When it rains, 80% times it makes thunderstorms. What is the probability that, in a particular day of rainy season, it will rain and it will thunderstorm?

Solution:

Let,

R= it will rain on that particular day

T= it will thunderstorm on that particular day

Probability Laws:

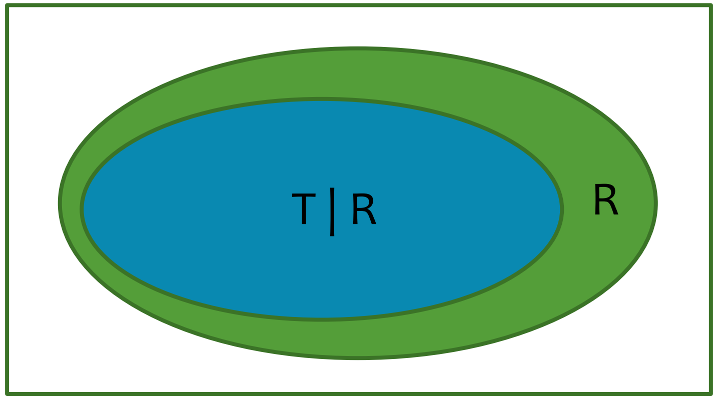
Multiplication Law

Solution(contd.):

Here, given that, $P(R) = \frac{70}{100} = 0.7$ and $P(T|R) = \frac{80}{100} = 0.8$

Therefore, the probability that, on that particular day of rainy season, it will rain and it will thunderstorm is-

$$P(R \cap T) = P(T|R)P(R)$$
$$= 0.8 \ast 0.7 = 0.56$$



Probability Laws:
Multiplication Law

Example 5:

Mr. Fahad and Mr. Khan has to tour abroad for their business frequently. Mr. Fahad tours 65% of the times in a year at abroad and Mr. Khan tours 50% of the times in a year at abroad. What is the probability that, on January 01, 2016, both Mr. Fahad and Mr. Khan will be at abroad?

Solution:

Let,

F= Mr. Fahad will be at abroad

K= Mr. Khan will be at abroad

Probability Laws:

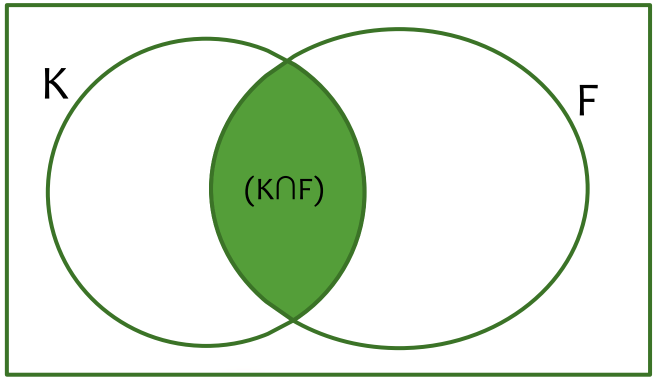
Multiplication Law

Solution(contd.):

Here, given that, $P(F) = \frac{65}{100} = 0.65$ and $P(K) = \frac{50}{100} = 0.5$

Therefore, the probability that, on January 01, 2016, both Mr. Fahad and Mr. Khan will be at abroad is-

$$P(K \cap F) = P(K)P(F)$$
$$= 0.5 * 0.65 = 0.325$$



Probability using contingency table

Example 7:

Below given a contingency table for Smoking status and Cancer status.

Smoking Status/ cancer Status	Cancer	Healthy	Total
Smoker	7860	1530	**9390**
Non-smoker	5390	11580	**16970**
Total	**13250**	**13110**	**26360**

1. What is the probability that a randomly selected person is a smoker
2. What is the probability that a randomly selected person has cancer?
3. What is the probability that a randomly selected person is both smoker and has cancer?
4. If a person is smoker, what is the probability that he also has cancer?

Probability using contingency table

Solution:

Let,

S= The person is smoker, N= The person is non-smoker

C= The person has cancer, H= The person is healthy

1. The probability that a randomly selected person is a smoker

$$P(S) = \frac{9390}{26360} = 0.356$$

Probability using contingency table

Solution **(contd.):**

2. The probability that a randomly selected person has cancer

$$P(C) = \frac{13250}{26360} = 0.503$$

3. The probability that a randomly selected person is both smoker and has cancer

$$P(S \cap C) = \frac{7860}{26360} = 0.298 = P(C|S)\,P(S) = \frac{7860}{9390} \times \frac{9390}{26360}$$

Probability using contingency table

Solution **(contd.):**

4. If a person is smoker, what is the probability that he also has cancer

$$P(C|S) = \frac{7860}{9390} = 0.837 = \frac{P(S \cap C)}{P(S)} = \frac{\frac{7860}{26360}}{\frac{9390}{26360}}$$